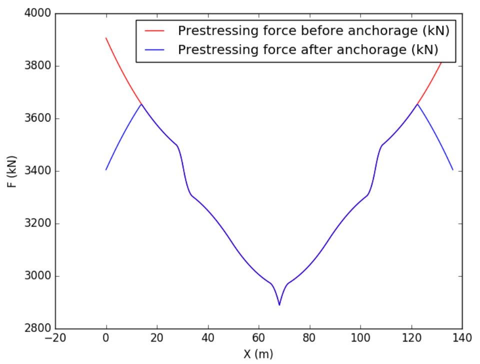
# Analysis of prestressed concrete structures with XC. Part I.- Immediate losses in prestress.




This article deals with the procedure for implementing the analysis of prestressed concrete structures in **XC** [a] . In particular, immediate losses of prestressing are considered, which include those due to friction at the tendon-concrete interface, the ones due to the slip of anchorages at the ends of the cable as well as the originated by the elastic shortening of concrete.
The geometry of the curved cables is interpolated from some characteristic points by using cubic splines, which allows to easily obtaining the curvilinear coordinate and its first derivative, required to evaluate the cumulated angular deviation involved in the prestressing loss due to friction. In order to enhance freedom when tracing the cables, nodes of tendon and those of concrete elements don't need to be coincident in space, instead, kinematic conditions are created between them to provide perfect adhesion.
Once the spline that adjusts the cable is obtained, the calculation of loss due to friction is straightforward.
To calculate the loss due to anchorage slip we must solve a problem with to unknowns into a single equation, for which an iterative method is used, by means of the Newton-Krylov solver implemented in the Scipy library.
Finally, two case studies are analyzed with **XC**.

[a] **XC** is a finite element program oriented to civil engineering. It is conceived as Open Source Software since we are developing it on the strong foundations of *OpenSees* and making heavy use of other OSS like Python, VTK and CGAL.

Ana Ortega, Luis C. Pérez Tato

## Geometry of cables

XC applies 3D splines to interpolate curved layouts of tendons, which is essential to obtain the curvilinear coordinate and the angle $\alpha$ involved in the formulas that evaluate the loss of prestressing.

We begin by defining, for each tendon, the representative points of its geometry by their global X,Y,Z coordinates. After that, we make use of the python-based library Scipy to interpolate the trajectory of the cable. The interpolation usually used is a cubic spline, that allows obtaining the derivative second continuous with polynomials of small degree. The precise calculation of the first derivative is required to evaluate the cumulated angular deviation $\alpha$ involved in the prestressing loss due to friction.

The cables are modeled with two-nodes linear elements, generally of type truss. A uniaxial steel material with isotropic strain hardening, that supports initial strain definition, is assigned to the cable elements. The concrete can be modeled with any type of shell or beam element. Kinematic conditions are created between the nodes of the cable and those of the concrete elements, to model a perfect adhesion between cable and concrete. In this way, the nodes of both members don't need to be coincident in space, which simplifies and allows more freedom to the cable's geometric design.

## Force variation diagram

The profile of tension along the cable is affected by friction at the tendon-concrete interface, by the slip of anchorages at the ends of the cable and by elastic shortening of concrete. These losses of prestressing force that occur during prestressing of the tendons and transfer of prestress to the concrete member, are classified as immediate versus the time-dependent losses, that include those due to shrinkage and creep of concrete and relaxation of the steel.

### Loss due to friction

We start by calculating the profile of tension in the cable after taking into account the loss due to friction during the streching of tendon in post-tensioned members (in pre-tensioning members this loss doesn't occur). The friction is generated due to the curvature of the tendon and the vertical component of the prestressing force. In addition, the streching has to overcome the wobble of the tendon, that refers to the unintentional deviation of the position of the tendon along the duct. The losses due to friction and wobble are grouped together under friction, that can be formulated in the folowing way:

$$F_c(s) = F_0 \cdot e^{-(\mu \cdot \alpha + k \cdot s)} \tag{1}$$

$F_c(s)$: force after friction loss along the curvilinear coordinate $s$ of the tendon.

$F_0$: prestress at the stretching end after any loss due to elastic shortening.

$\mu$: coefficient of friction between the curved tendon and its sheating, expressed in $rad^{-1}$.

$\alpha$: cumulative angular deviation, in $rad$.

$k$: wobble coefficient or coefficient for wave effect, per unit length of cable.

The cumulative angular deviation $\alpha$ along the cable is obtained from the first derivative of the interpolated 3D spline, by adding the angles between the tangent vectors to the curve at the successive interpolated points.

### Loss due to anchorage slip

In a post-tensioned member, the tendon reduces its length when the anchorage system moves before it settles on the concrete and, as a consequence, there is a loss of tension. As the tendon shortens, a reverse friction occurs, so that the effect of anchorage slip is present up to a certain length, beyond which, there is no loss of tension. So, we have a problem with two unknowns (see fig. 1): the setting leng $l_{set}$ and the function $F^*(s)$ that represents the force after anchorage slip:

$$\Delta = \int_0^{l_{set}} \epsilon ds = \frac{1}{E_p A_p} \int_0^{l_{set}} [F_c(s) - F^*(s)] ds \tag{2}$$

where,

$\Delta$: anchorage slip, depending on the type of anchorage system. Generally is a datum provided by the manufacturer.

$E_p$: elastic modulus of the prestressing steel.

$A_p$: cross-sectional area of the tendon.

$l_{set}$: length of tendon affected by the prestressing loss due to the anchorage slip.

$F_c(s)$: force variation diagram after friction loss.

$F^*(s)$: force variation diagram after loss due to anchorage slip.

To find the roots of the equation, an iterative method is used, by means of the Newton-Krylov solver implemented in the Scipy library.

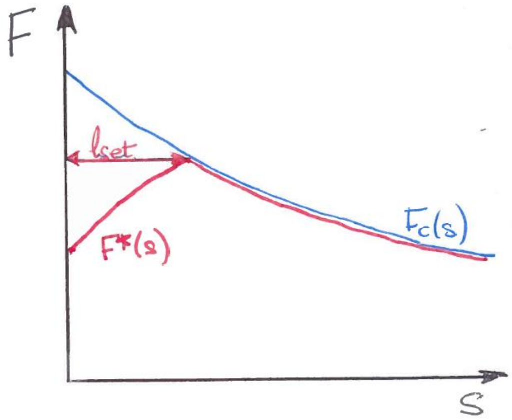



Figure 1: Loss due to anchorage slip

Several typical cases can arise:

1. The cable is stretched from one end and the loss of tension is localized in the zone of anchoring (fig. 2).

2. If the cable is curved and sufficiently short, it can happen that the resulting $l_{set}$ is larger than the length of the cable (fig. 3). In this case, the prestressing loss due to anchorage slip applies everywhere and $F^*(s)$ is calculated so that the surface between the curves $F_c(s)$ and $F^*(s)$ equals $\Delta \cdot E_p \cdot A_p$.

3. If the cable is stretched from both ends, let us call $F_1(s)$ the distribution of tension after friction loss calculated as if stretching only from the first end and $F_2(s)$ the tension as if the force was applied only to the second anchoring (fig. 4 and 5). The value that must be retained at any point of the cable as initial tension is $F(s) = \max\{F_1(s), F_2(s)\}$ .

4. Finally, if $l_{set}$ is larger than the length of the cable and it is prestressed from both anchorages, the following procedure is applied:

   - The initial tension $F_1^*(s)$ is calculated, taking account of friction and achorage slip losses, as if the tension was applied only to the first anchoring.
   - $F_2^*(s)$ is likewise calculated as if stretching only from the second anchorage.
   - The initial tension $F(s) = \min\{F_1^*(s), F_2^*(s)\}$ is retained for each point of the tendon.

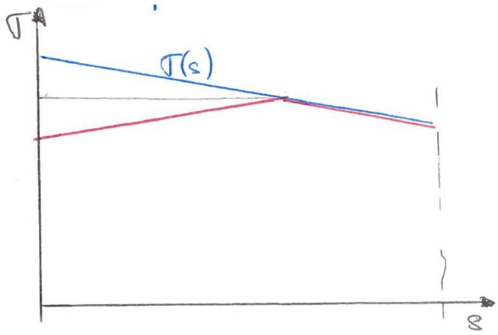



Figure 2: Case 1 of loss due to anchorage slip

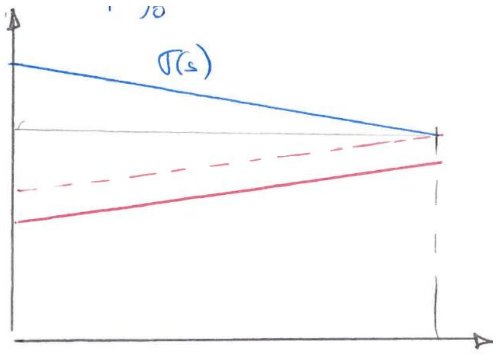



Figure 3: Case 2 of loss due to anchorage slip

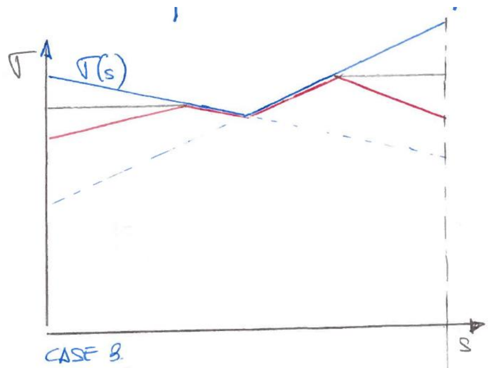



Figure 4: Case 3 of loss due to anchorage slip

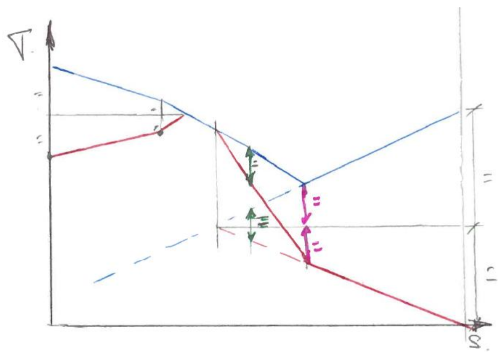



Figure 5: Case 3 of loss due to anchorage slip

## Loss due to instantaneous elastic shortening of concrete

In post-tensioned members, if there is only one tendon there is no loss because the applied prestress is recorded after the elastic shortening of the member. For more than one tendon, if the tendons are stretched sequentially, there is a loss in a tendon during subsequent stretching of the others. To model this phenomenon in XC it is necessary to represent all the prestressing stages. The birth and death of elements is very useful for this purpose. For the case of the first tendon, the loss of prestressing that occurs in the model due to elastic shortening is corrected by the re-stretching of the tendon in one or several iterations until equilibrium is reached.

## Examples

Figures 6 to 8 are related to an example of calculation taken from reference [2]. It deals with a four span (30.4 - 37.8 - 37.8 - 30.4 m) continuous bridge girder, post-tensioned with a tendon with $f_{pk} = 1860 MPa$, that is simultaneously stressed up to 75% $f_{pk}$ from both ends and then anchored. The tendon properties are $A_p = 2800\ mm^2$, $E_p = 195000\ MPa$, $\mu = 0.20$ , $k = 0.0020\ m^{-1}$, and the anchorage slip $\Delta = 6\ mm$.

Figure 6 shows the geometry of the tendon, interpolated by means of a cubic spline. The inflection points of the curved tendon are an input to the interpolation algorithm and, once obtained the spline function, a total of 300 points are interpolated, at which the curvilinear coordinate, derivatives, prestress, . . . , will be calculated. Figure 8 shows the force variation diagrams along the tendon before and after anchorage issued by the XC model.

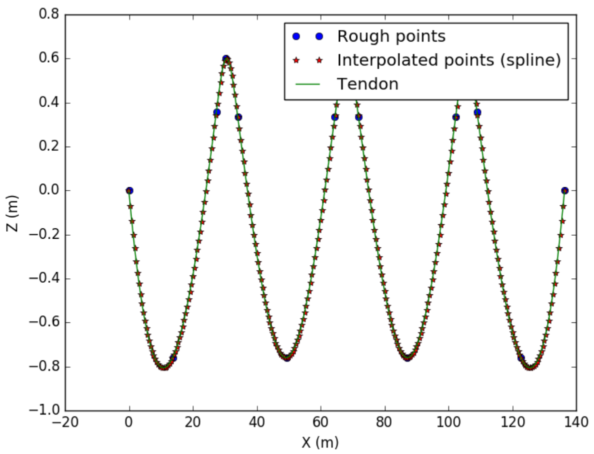



Figure 6: Example 1: interpolation of the tendon trajectory

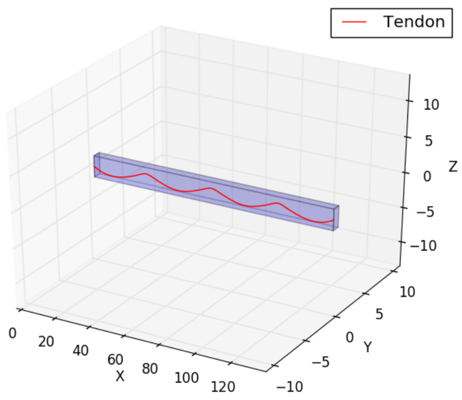



Figure 7: Example 1: 3D view of the prestressed beam

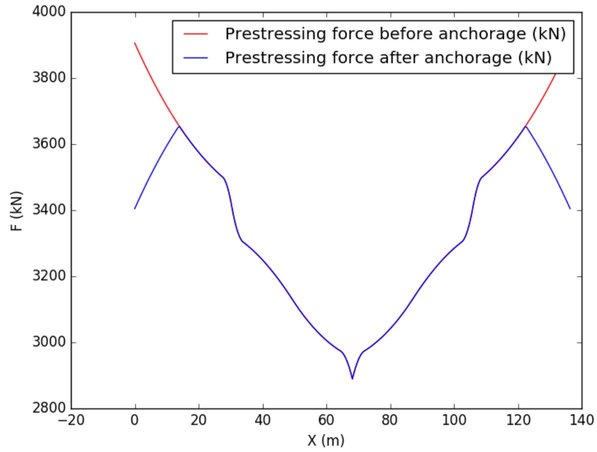



Figure 8: Example 1: force variation diagram

The second example (figures 9 to 18) deals with a post-tensioned beam $260 \times 1630\ mm^2$ $(b \times h)$ spanning over 20 m which is stressed by successive tensioning and anchoring of 2 cables. The cables follow a parabolic trajectory with eccentricity 0 at both ends and 558 mm at mid span. Characteristics of materials, cables and prestressing system are summarised below:

| | |
|---|---|
| Modulus of elasticity of concrete $E_c =$ | 2.6e10 Pa |
| Modulus of elasticity of prestressing steel $E_p =$ | 1.95e11 Pa |
| Coefficient of Poisson of concrete $\nu_c =$ | 0.2 |
| Specific mass of concrete $\rho_c =$ | 2500 $kg/m^3$ |
| Yield stress of the steel $f_y =$ | 1171e6 Pa |
| Area of tendon cross-section $A_{ps} =$ | 1.425e-3 $m^2$ |
| Initial stress in each tendon $f_{pi} =$ | 620e6 Pa |
| Friction coefficient $\mu =$ | 0.20 |
| Wobble coefficient $k =$ | 0.0020 $m^{-1}$ |
| Anchorage slip $\Delta =$ | 6e-3 m |

Calculation is carried out in three phases:

- Phase 1: Modelling of beam and tendons, prestressing of cables, calculation of profile of tension after immediate losses, death of cable 2. Results are shown in figs. 9 to 11.

- Phase 2: applying of beam selft-weight load (figs. 12 to 14).

- Phase 3: birth of prestressed elements of cable 2. (figs. 15 to 18).

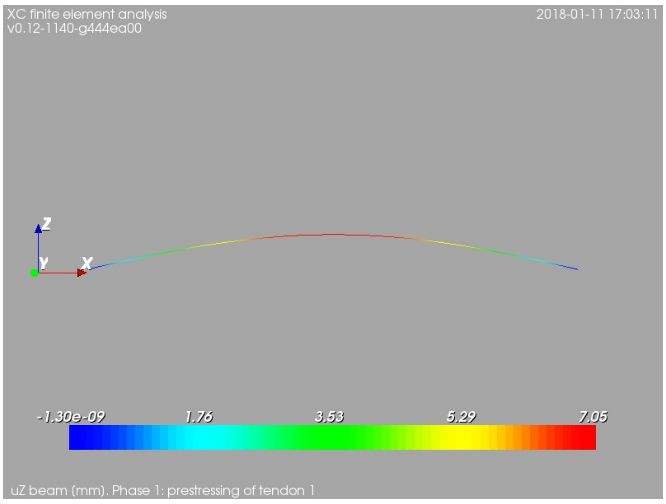



Figure 9: Example 2. Phase 1: prestressing of tendon 1. Vertical displacements.

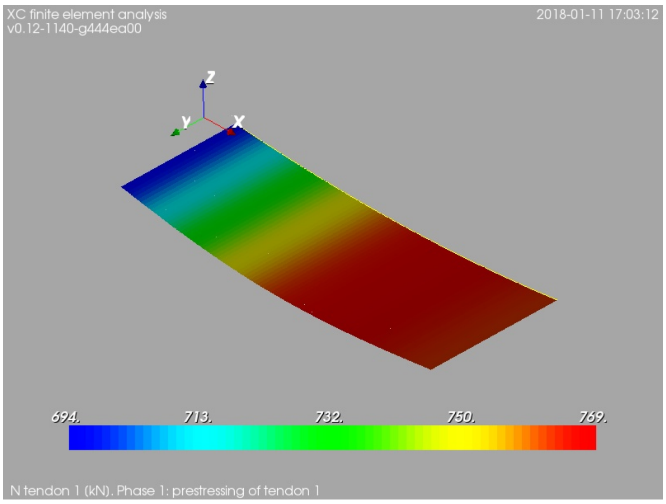



Figure 10: Example 2. Phase 1: prestressing of tendon 1. Axial force in tendon.

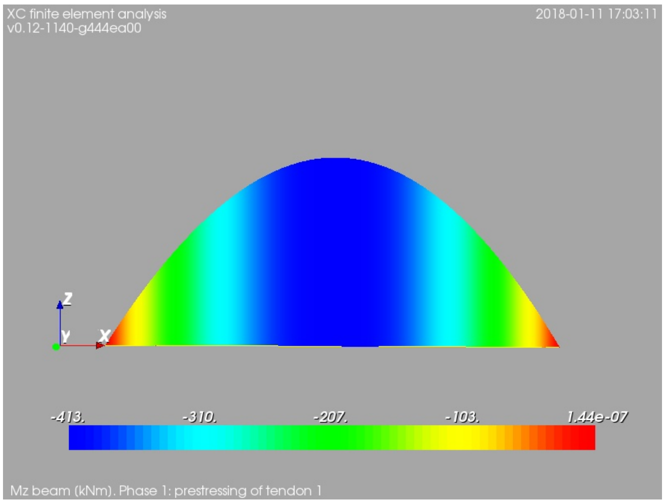



Figure 11: Example 2. Phase 1: prestressing of tendon 1. Bending moment in beam.

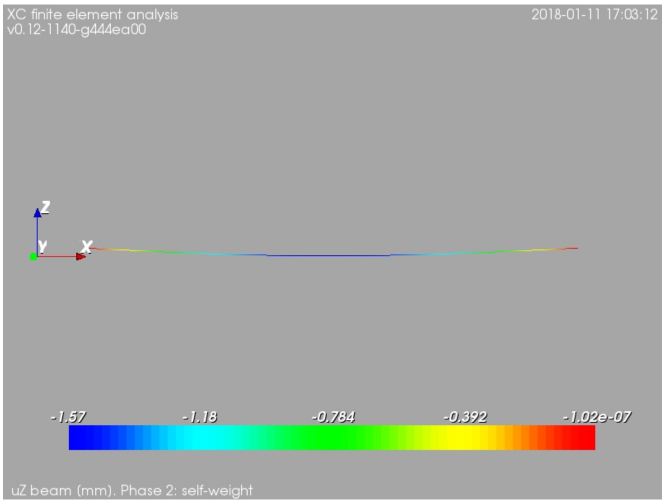



Figure 12: Example 2. Phase 2: self-weight. Vertical displacements.

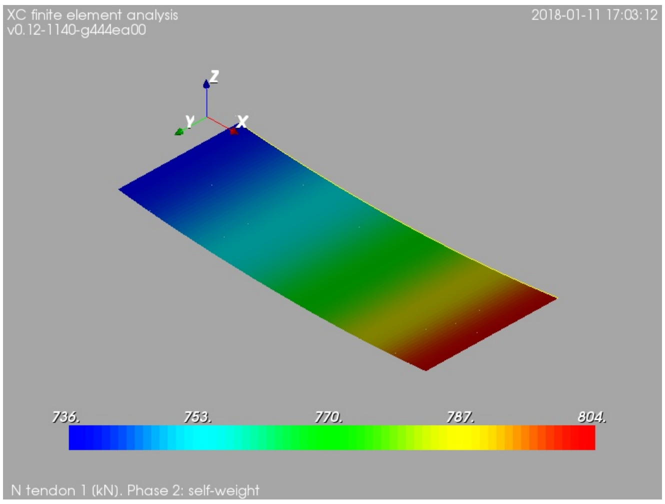



Figure 13: Example 2. Phase 2: self-weight. Axial force in tendon.

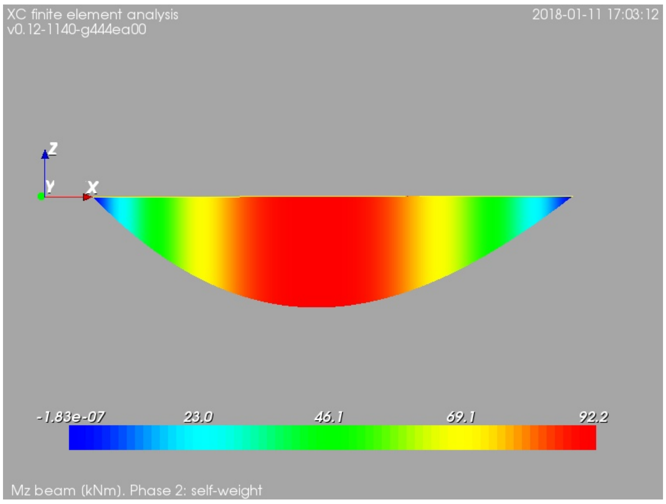



Figure 14: Example 2. Phase 2: self-weight. Bending moment in beam.

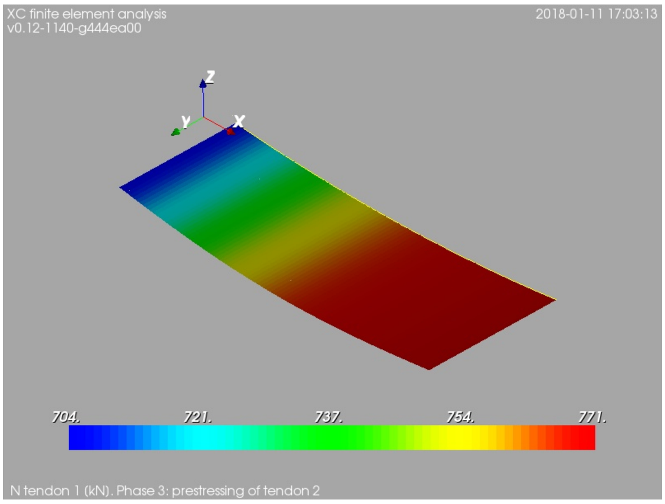



Figure 17: Example 2. Phase 3: prestressing of tendon 2. Axial force in tendon 1.

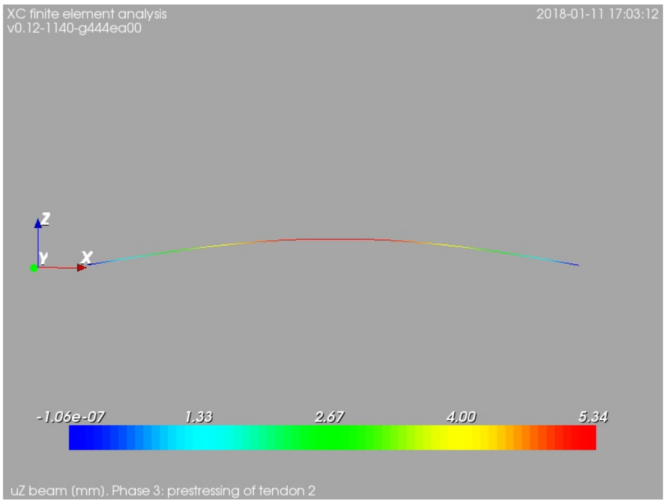



Figure 15: Example 2. Phase 3: prestressing of tendon 2. Vertical displacements.

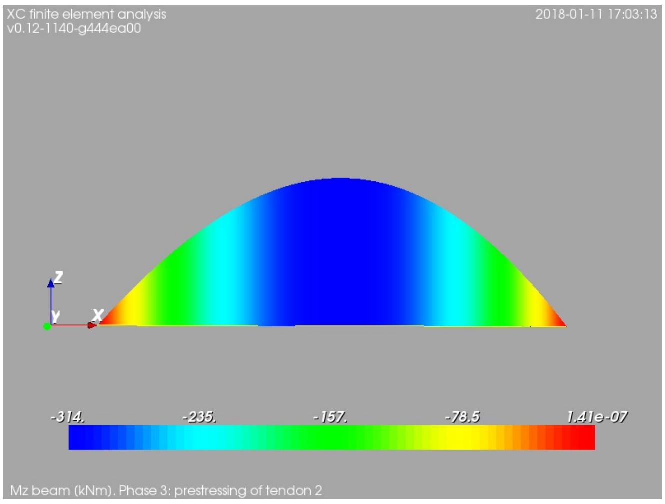



Figure 18: Example 2. Phase 3: prestressing of tendon 2. Bending moment in beam.

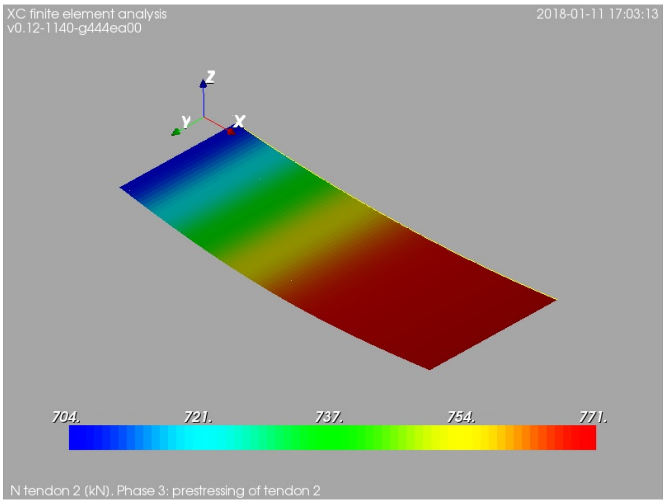



Figure 16: Example 2. Phase 3: prestressing of tendon 2. Axial force in tendon 2.

# References


[1] Sylvie MICHEL-PONNELLE. Modélisation des câbles de précontrainte. *Code-Aster*, R7.01.02, 2013.

[2] Dr. Amlan K. Sengupta and Prof. Devdas Menon. Prestressed concrete structures.